

THE LEAST- ACTION PRINCIPLE AND ITS APPLICATIONS IN QUANTUM MECHANICS

Vu huy toan
Vietnamese Academy of Science and Technology
18 Hoang Quoc Viet str., Cau giay dist., Hanoi, Vietnam
Tel: 84-04-5729971, Fax: 84-04-5657573
E-mail:vuhuytoan@hn.vnn.vn

abstract

The least-effect principle in mechanics is known with the name of “Hamilton Principle”. The existence of least-effect value proposed by Max Planck has been widely applied in quantum mechanics. By mean of exactitude the concept of “action” in combination with the least-effect principle and Planck constant, the author has set forth a new principle called as “least–action principle”. The application of this principle in the quantum mechanics leads to the compulsory review of fundamental particles’ motion and interaction. Not only energy and angular momentum but also the very momentum, velocity, angle of motional deflection as well as time are discrete too. The electron’s condition of orbit quantum in atom has been re-estimated. Finally, the estimation of any motion which could only be deflected with limited “angle quantum” has indicated the non-existence of any “physical wave”; the fundamental particles, in deed, has no “wave property” but only “similar to wave manifestation”.

Keywords: wave property, physical wave, least action, least effect, angle quantum.

I. Introduction.

Returning to the classical mechanics in the Era of “*Theory of Everything*” with “*membrane*”, “*super string*”, “*super symmetry*”,... is a sterile work of in the thought of many people. But, how awkward it is, the Nature always leads us to the challenge which never has only unique solution. There existed problems we could only realize not correction after hundreds of years. Therefore, sometime the returning to “*too old*” issues never is superfluous. Following is perhaps one of these cases. As we know, it is supposed that the thermo-radiation energy is not continuous but discrete by each small portion:

$$\varepsilon_o = hf, \quad (1)$$

with f as radiation frequency and h as any coefficient of rate, Max Planck found out the suitable with experiment formula of absolute blackbody radiating intensity [1]. The h coefficient is later called as “Planck constant”. In the macroscopic world, the Planck constant plays a role as threshold of least-momentum when Bohr set forth the electron’s condition of orbit quantum in atom:

$$M_n = m_e v_n r_n = n(h/2\pi), \quad (n = 1, 2, 3, \dots) \quad (2)$$

In the later time, h entered into quantum field theory as an unavoidable parameter in the Schrodinger equation, in expressions of fundamental particle angular momentum and magnetic moment... That proved that the Planck’s

prediction has been determined firmly. In case of dimension, it fits to the dimension of “effect” in the *least-effect principle* (LEP) or also called “*Hamilton principle*” [2,3]. The H *effect* function specialized the mechanical motional energy state of mechanical system. H can be written in two ways:

$$+ \text{Hamilton- Ostrogratsky:} \quad H = \int_{t_0}^{t_1} L dt, \quad (3)$$

$$+ \text{Maupertuis- Lagrange:} \quad H = \int_{t_0}^{t_1} 2Edt, \quad (4)$$

of which L and E – Lagrangien and kinetic energy of mechanical system respectively; t_0 and t_1 – initial and finishing time of the mechanical system’s motion from point A to point B respectively. According to LEP, in the feasible motion of mechanical system, the actual motion is the one that makes *effect* function minimize (more precisely extremum), it means to turn the isochronal variation of *effect* function to zero: $\delta H = 0 \rightarrow H = H_{\min}$; of which H_{\min} is the smallest *effect* function from all feasible *effect* functions of the mechanical system when it moves from point A to point B. These points are fully correct with macroscopic world. In the microscopic world, due to location unstable property or *wave-particles amphoteric* of fundamental particle, LEP are no longer to be applied. To say tingly, it is the inexistence of particles but *particle-wave entity* (a structure of location instability) that could be the cause and if the particle remains as it is and its difference is to have “*wave property*”, thus there existed no reason not to apply LEP.

The problem is set up as that in case of accepting the hypothesis of h as *least-effect* quantum simultaneously with LEP, then could the mechanical energy state (not only angular momentum as in Bohr’s hypothesis) of physical objects be discrete like thermal energy? And if happened, then how are their motion as well as interaction? Of course, in this case, the “*wave property*” of particles must be received as necessary consequence. We shall check this hypothesis.

II. The least-action principle.

II.1.Fundamental concepts.

a/ Action among physical objects.

In documents, including original documents in English, people usually use confusedly two concepts of “*action*” and “*effect*”. Hereafter, we try to distinguish them from the orthodox mechanical point of view.

Supposing that a physical object has mass m moving under the action of an unchangeable force F from point A to point B in duration from t_0 to t_1 . As already known, the concept of “*effect*” defined according to (3) and (4) is the actual energy state of physical object is that already occurred in that time interval. Replacing kinetic energy expression of physical object under consideration into (4), we have:

$$H = \int_{t_0}^{t_1} mv^2 dt = \int_{t_0}^{t_1} mv_0^2 dt + 2 \int_{t_0}^{t_1} ma(v_0 t + at^2/2) dt = \int_{t_0}^{t_1} 2T_0 dt + \int_{t_0}^{t_1} 2FS(t) dt = H_0 + H_t, \quad (5)$$

where

$$H_0 = \int_{t_0}^{t_1} 2E_0 dt, \quad (6)$$

$$H_t = \int_{t_0}^{t_1} 2FS(t) dt = \int_{t_0}^{t_1} 2A(t) dt. \quad (7)$$

It may be realized immediately that: $A(t) = FS(t) = ma(v_0 t + at^2/2)$ is the work of action force $F = ma$, with which m and a - mass and motional velocity of physical object under action of a force F correspondingly. It is clearly that, if there is no action force ($F = 0$), then $H_t = 0$ and $H = H_0$ - the *action* component of physical object in the free state (uniform motion at velocity v_0). Component H_t is the very *result* of action force F on physical object under consideration. In other words, in order to have “*effect*” H_t , requires corresponding “*action*”, in sign of D .

Whereas, because the energy transferring speed is always limited by the light velocity $c \approx 3 \cdot 10^8$ m/s, the exchanging energy between physical objects' time always differs from zero. Furthermore, the transitional process of them from this energy state to other also requires a limited time interval. That's why, the *action* D needs to overpass *effect* H_t a certain time interval of τ :

$$D = \int_{t_0 - \tau}^{t_1 - \tau} 2E(t) dt, \quad (8)$$

where $E(t)$ – energy exchanged between 2 physical objects leading to the change of energy state, it means to have appearance of *effect* H_t . In the ideal case, the entire energy $E(t)$ transforms into work $A(t)$, we shall have:

$$H_t = D. \quad (9)$$

Normally, while $E(t)$ is very difficult to be defined, then basing on the equation (9) we may only define work $A(t)$ or the energy change ΔE of physical object in replacement of $E(t)$. At that point, in deed, we define *effect* H_t , and not *action* D .

b/ Least effect and least action.

As recognized, there impossibility exists any *effect* smaller than the *least-effect* h . This also means that the *effect* function H on (3) and (4) is not continuous and can only be a multiple number of h :

$$H = nh \quad (n = 1, 2, 3, \dots) \quad (10)$$

Whereas, any occurring physical process always goes along with the energy exchange. That exchange occurs on each small portion. Thus, responding to the *least effect* h is the *least action* d . In other words, there exists a threshold of *least action*, if smaller, then the considered physical object fails to have state changed. That threshold is called "*least action*"- d . Proposed that the exchanging efficiency $\eta = 1$, then $d = h$. It means the entire *action* turns into *effect*.

II.2. Content.

As known, the study on fundamental particle motion in classical mechanical scope failed fully. The fundamental reason is that in this scope, the *effect* function has become possible to be compared with the *least effect* h , it is mean the expression (10) has an effect. The problem may be different if we put this into account.

Recognizing the causality between *action* D and *effect* H_t on (6) and paying attention to (10), we may describe as follows:

"In order a physical object may changes its energy state, the action against it must not be smaller than the least effect".

$$D = \int_{t_0 - \tau}^{t_1 - \tau} 2E(t)dt \geq h. \quad (11)$$

We call this as "***least-action principle***" (LAP). Therefore, different from LEP (least-effect principle), LAP lays condition on *action* D (*cause*) but not on *effect* H (*consequence*). In addition, that condition does not aim at optimizing various types of *action* functions, but in contrast, it makes it possible for any *action*. The inequation (11) is also called *effect condition* (EC). In the time interval from $(t_0 - \tau)$ to $(t_1 - \tau)$, and $E(t) = E = \text{const}$, then (11) can be written as:

$$D = 2E(t_1 - t_0) = 2E\Delta t \geq h. \quad (12)$$

From (12), it can be seen that if energy E is very great, then the *action time* Δt can be selected small enough to have "*effect*". On the contrary, if the exchanging energy is too small, then the *action time interval* Δt needs to be great enough. While, this time interval could not be too great and usually be limited by some conditions:

+In case, the live time of object under consideration is limited by τ_1 , then $\Delta t \leq \tau_1$;

+In case, the feasible time interval for energy exchanging possibility of physical objects is τ_2 , then $\Delta t \leq \tau_2$;

+In case, the moving under action object has period of T, then $\Delta t \leq T$ because, if in the time interval equal to 1period T, the exchange is “without effect”, thus although the *action* may prolong in the successive periods, the “without effect” still remains.

III. Consideration of some phenomenon on the basis of LAP.

III.1.The particles motion under action of a force.

In the classical mechanics as well as in the relative mechanics, one all use the Newton 2 Law in the form of *velocity of the impulse p change of particle by the action force F against it* [1,5]:

$$F = d(mv)/dt \quad \text{hay} \quad F = dp/dt, \quad (13)$$

of which m and v - mass and motional velocity of particle correspondingly. If the motional velocity is not great in comparison with light velocity, then m can be possibly considered as constant, and thus (13) could be written in the form of:

$$mdv/dt = ma = F, \quad (14)$$

where $a = dv/dt$ – called as motional instantaneous acceleration of the particle.

Both (13) and (14) indicate that under the action of force $F = \text{const}$, then the particle motional velocity will very continuously. This point is entirely not in conformity with LAP. To make it clear, let’s consider the concept of derivative of velocity v on time t:

$$a = dv/dt = \lim_{\Delta t \rightarrow 0} (\Delta v/\Delta t) = \lim_{\Delta t \rightarrow 0} a_{tb}, \quad (15)$$

of which, $a_{tb} = \Delta v/\Delta t$ – medium acceleration, and a is called instantaneous acceleration. As the energy exchanged in the time interval Δt is limited, then, if $\Delta t \rightarrow 0$, multiplication of E. Δt also advance to 0 and LAP is violated. Therefore, if recognizing *effect condition* (11), the concept of acceleration estimated on (15) is of no significance because Δt can not be smaller that the value: $h/E \neq 0$ and $\Delta t \rightarrow 0$ fails to be talked about. In other words, (13) or (14) are mathematical expressions imitating rudely the real motion of particle. Then, how is the so-called “real motion” of particle under action of a force F? To simplify it, we only consider the case of $m = \text{constant}$ and replace the motional equation bearing derivative of velocity (14) by that with limited numbers:

$$ma_{tb} = m(\Delta v/\Delta t) = F, \quad \text{or} \quad a_{tb} = \Delta v/\Delta t = F/m = \text{const.} \quad (16)$$

In case that prior to the action force F at time of $t = t_0 = 0$, the particle is in the state of rest $v_0 = 0$, then after having action force F in the time interval of $\Delta t_1 =$

$t_1 - t_0 = t_1$, satisfying the *effect* condition (12), the particle will move at velocity v_1 . The energy received by particle is equal to its kinetic energy difference in 2 states.

$$\Delta E_1 = E_1 - E_0 = mv_1^2/2 - mv_0^2/2 = mv_1^2/2. \quad (17)$$

Replacing (17) into (12), we have: $2 mv_1^2 \cdot \Delta t_1 / 2 \geq h$. (18)

In other way similar to (16), we may write: $a_{tb} = \Delta v_1 / \Delta t_1 = F/m$, (19)

of which $\Delta v_1 = v_1 - v_0 = v_1$. Taking Δt_1 from (19) and inserting into (18), then with change:

$$\Delta v_1 = v_1 \geq \sqrt[3]{ha_{tb}/m} = \sqrt[3]{hF/m^2}. \quad (20)$$

Replacing (20) into (19), then taking out Δt_1 , we have:

$$\Delta t_1 = t_1 \geq \sqrt[3]{hm/F^2}. \quad (21)$$

With the particle motional problem deduced from (19) in general:

$$a_{tb} = \Delta v_n / \Delta t_n = F/m, \quad (22)$$

of which $\Delta v_n = v_n - v_{n-1}$; $\Delta t_n = t_n - t_{n-1}$; $n = 1, 2, 3, \dots$ - positive prime number. The relation between v_n and t_n can be defined by forming the rate:

$$(\Sigma \Delta v_n) / (\Sigma \Delta t_n) = v_n / t_n. \quad (23)$$

In other way, from (22), taking out Δv_n , then inserting into (23), with changes, we have:

$$v_n = a_{tb} t_n. \quad (24)$$

Expressions of (22) and (24) show us the relation between n of discrete velocity value v_n and not discrete time t_n . In any case, we can write:

$$2\Delta E_1 \Delta t_1 = 2\Delta E_2 \Delta t_2 = \dots = 2\Delta E_n \Delta t_n = h. \quad (25)$$

Replacing: $\Delta E_n = E_n - E_{n-1} = m(v_n^2 - v_{n-1}^2)/2$

and Δt_n from (22) into (25), with changes, we have:

$$m(v_n^2 - v_{n-1}^2)(v_n - v_{n-1})/a_{tb} = h. \quad (26)$$

Replacing v_1 from (20) into (26), then with changes, we have $(n-1)$ equations:

$$v_n^3 - v_{n-1}v_n^2 - v_{n-1}^2v_n + v_{n-1}^3 - v_1^3 = 0. \quad (27)$$

Solving (27) we have $(n - 1)$ velocity values from v_2 to v_n . Replacing v_n from (24) into (27) with changes, we have $(n - 1)$ equations corresponding to variable time from t_2 to t_n :

$$t_n^3 - t_{n-1}t_n^2 - t_{n-1}^2t_n + t_{n-1}^3 - t_1^3 = 0. \quad (28)$$

The graph presenting the velocity changes depending on particle motional time is shown in the figure 1a. It can be clearly seen that the particle may only move in the manner of “*recoil steps*” with gradually increase of velocity according to each “*step*”. If counting the particle motional delay after time interval Δt_1 , then the distance of a particle could pass after each time interval Δt_{n+1} will be: $\Delta S_n = v_n \Delta t_{n+1}$. Thus, the particle motional equation is a broken line:

$$S_n = \Sigma \Delta S_n = \Sigma v_n \Delta t_{n+1}. \quad (29)$$

Replacing v_n from (24) into (29), we have:

$$S_n = a_{tb} \Sigma t_n \Delta t_{n+1} = a_{tb} \Sigma t_n (t_{n+1} - t_n). \quad (30)$$

For comparison, we write expression of distance the particle has gone during the time t on normal way: $S(t) = at^2/2$. At the time $t = t_n$, then $a = \infty$, and at time of $t \neq t_n$, then $a \equiv 0$. From this point, it can be seen that the concept of instantaneous acceleration is here of no significance.

III.2. Wave property of fundamental particles.

The concept of *physical wave* has been pointed out by de Broglie and verified by Davission and Germer through experiments [6]: each particle combines with a wavelength:

$$\lambda = h/p = h/mv. \quad (31)$$

The reality is that the *wave property* and *particle property* can never be presented simultaneously and the *wave property* can only be presented when meeting obstacle on the way like narrow slits, small holes or crystal of some substances playing a role of grating.

Another reality is that the atoms or molecules may have mutual interaction on the distances of approximate 10 times of their diameter [1]. This distance is also called as *effect radius* R . Thus, wouldn't the particles be effected when entering into these molecules' *effect radius*? Of course, this action force is very small. It is of nothing in case of particles with great inertia. But in case of particles with small inertia, how will their motion be? According to the EC, if after the time interval Δt_1 , particle receives *action* D_1 equal or greater than *least effect* h , it will change direction of their motion on a defined *angle quantum* α_{01} . If after that, it does not receive successive *action*, then it

remain in the same motional direction. If the successive *action* $D_2 \geq h$, it will change motional direction at second time at deflected angle α_{02} .

So on and so forth, after passing obstacles, a primary beam of particles is separated into smaller beams deflecting with initial direction at different angles quite defined $\alpha_{01}, \alpha_{02}, \alpha_{03}, \dots$. If we place a screen on a distance far from that, we shall receive a picture of “*diffraction*” like the same wave do. In reality, the process may happen in more complex way. The particle must go through the field not only of a molecule but also that of many, many molecules, however, this is not important. The key point here is that the particle’s direction can not be deflected at any angle but on a defined and limited angle quantum. The result is that after many times of deflection, the ***total deflected angle is merely limited and defined.***

For quantitative, we shall consider the particle’s deflection under particular *action* of molecule on the curtain slit edges of A and B separately. From symmetry at first we only consider particle’s deflection on the slit edge A of curtain. Suppose that particles with mass m , motional velocity v falling into the *effect* radius of molecules on the slit edge A of curtain at points of a, b, c, d, \dots flying to corresponding points of $a', b', c', d' \dots$ (see fig.1).

At first, in case of particles flying on way aa' and deflecting only one time with an angle quantum α_a , the energy received by them in time interval $\Delta t_a = aa'/v = S_a/v$ equal to:

$$\Delta E_a = E_a - E_{a'} = mv^2/2 - mv'^2/2 = mv^2(1 - \cos^2\alpha_a)/2 = mv^2\sin^2\alpha_a/2. \quad (32)$$

With attention to (9) and (10), we may write:

$$D_1 = H_{t1} = 2\Delta E_a \Delta t_a = mv S_a \sin^2\alpha_a = h. \quad (33)$$

or:
$$S_a \sin^2\alpha_a = h/mv = h/p. \quad (34)$$

In case of particles flying on way of bb' , there are two times of deflections.

The first time, while flying from b to b_1 , at the time interval Δt_{b1} , it receives energy ΔE_{b1} . Because the potential field on way of bb' is stronger than that in aa' , then, on principle, at the same time interval, particles must receive more energy. According to LAP, reaching the time when satisfied (12), the particles may have already deflected motional direction at angle α_{b1} . The received by them energy at the time interval of $\Delta t_{b1} = bb_1/v = S_{b1}/v$ is: $\Delta E_{b1} = mv^2/2 - mv_{b1}^2/2 = mv^2(1 - \cos^2\alpha_{b1})/2 = mv^2\sin^2\alpha_{b1}/2$. Similar to (34), we may write:

$$D_1 = H_{t1} = 2\Delta E_{b1} \Delta t_{b1} = mv S_{b1} \sin^2\alpha_{b1} = h. \quad (35)$$

The second time, while flying on the way $b_1b' = S_{b_2}$ at the time interval of $\Delta t_{b_2} = b_1b'/v = S_{b_2}/v$, it is deflected at angle α_{b_2} due to the reception of energy equal to: $\Delta E_{b_2} = mv^2/2 - mv_{b_2}^2/2 = mv^2 \sin^2 \alpha_{b_2}/2$. Similar to (34), we may write:

$$D_2 = H_{t_2} = 2\Delta E_{b_2} \Delta t_{b_2} = mv S_{b_2} \sin^2 \alpha_{b_2} = h. \quad (36)$$

Adding of (35) with (36) on corresponding members and rewriting under the form:

$$S_{b_1} \sin^2 \alpha_{b_1} + S_{b_2} \sin^2 \alpha_{b_2} = 2h/mv = 2h/p.$$

In case of particles moving on way of cc' and dd' with 3 and 4 times of direction deflecting, we may write similarly:

$$S_{c_1} \sin^2 \alpha_{c_1} + S_{c_2} \sin^2 \alpha_{c_2} + S_{c_3} \sin^2 \alpha_{c_3} = 3(h/mv) = 3(h/p).$$

$$S_{d_1} \sin^2 \alpha_{d_1} + S_{d_2} \sin^2 \alpha_{d_2} + S_{d_3} \sin^2 \alpha_{d_3} + S_{d_4} \sin^2 \alpha_{d_4} = 4(h/mv) = 4(h/p).$$

Generally with any "k" particles with n times of direction deflecting, we have:

$$\sum_{k=1}^n S_{k_n} \sin^2 \alpha_{k_n} = n(h/mv) = n(h/p). \quad (37)$$

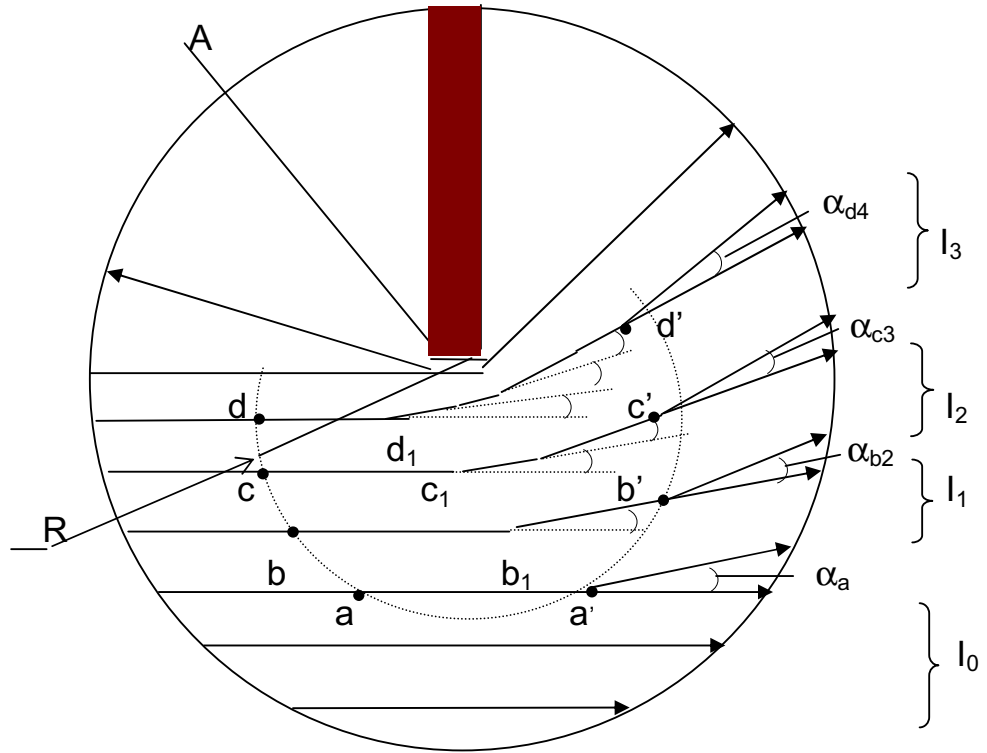


Fig.1. Diffraction of electrons beam flying among slit curtain.

The left member of (37) is parameters relating to the potential field in the curtain slit, and the right member of (37) only relates to particles, in more concrete way to particles' impulse p . When the width of the curtain slit similar to *effect* radius R , then particles are acted at the same time by both two edges of A and B. Because of that the field of atoms on these two edges producing in slit always change direction randomly, thus, some times having the same direction, and some times have opposite direction and even eliminating one another, that's why it is impossible to estimate the direction deflecting of actual particles. However, the number of angle quantum n and these very angles α_{kn} are limited and defined.

We can fully provide $h/p = \lambda$ similar to (31). This is the one that is called as "de Broglie wavelength", but in fact there is not existing any wave. The unique existence here is still particle not more or less. By such the similar mean, we may receive picture of "*interference*" when the particles beam go through two small holes of the curtain or the picture of "*diffraction*" on the crystal of some substances.

In case of particles with too great impulse, the potential field energy of molecules on slit edge of curtain is not enough to slant direction unless there is direct contact. That's why, they do not have '*wave property*'. In case of particles without electricity as a neutron, how can they be direction biased in the potential electric field? The answer is that the neutron does not mean to be without electricity but be neutralized in electricity similar to atoms, but as its dimension is too small, thus, it can be present only at a distance very near to it.

Therefore, it is correct in projection as "*wave property*" is the direct consequence of LAP application. From this point, it can be understood why the fundamental particles' "*wave property*" can be present when facing obstacle but not in the free space. Finally, to say in firm that the particles have no "*wave property*" because of that there is here no "*addition of amplitude in same phase or elimination of amplitude in opposite phase*" as a real wave [1]. We may only say that "*particles have a similar wave manifestation*"! Therefore, there not exists the so-called "*wave-particle amphoteric*" too!

III.3.The orbit angular momentum of electron.

Supposed that the electron is at point A, far from nucleus a distance of r_n and moves at velocity v_n to point B as shown in figure 2. Supposed that the electron moves on orbit which is enclosed way. In this case we provide the sign to quantity with index of "n", being quantity corresponding to orbit "n" of electron according to the orbit order from internal to external side. The smallest orbit r_1 corresponding to $n = 1$. The electron's motion in atom under the action of Coulomb force can only be directionally biased when meeting EC (12). The energy for the first deflected angle is also defined on (32), and the time interval of receiving energy is equal to:

$$\Delta t_n = AB/v_n = 2r_n \sin(\alpha_{on}/2) / v_n.$$

This first *effect* is: $H_{to} = 2\Delta E_n \cdot \Delta t_n = 2m_e v_n r_n \sin^2 \alpha_{on} \sin(\alpha_{on}/2) = h.$

In order to receive the enclosed orbit which an equal polygon inscribed to cycle at radius r_n , then after k_n *angle quantum step* α_{on} , the electron must return right to point A, it means:

$$k_n \cdot \alpha_{on} = 2\pi. \quad (38)$$

While, orbit “n” must have k_n *least effect* quantum h:

$$H_{tn} = k_n H_{to} = k_n 2\Delta E_n \Delta t_n = 2\Delta E_n T_n = 2k_n m_e v_n r_n \sin^2 \alpha_{on} \sin(\alpha_{on}/2) = k_n h, \quad (39)$$

of which $T_n = k_n \Delta t_n$ - rotation on “n” orbit period of the electron. From here we deduce the expression of orbit angular momentum:

$$M_n = m_e v_n r_n = h / (2 \sin^2 \alpha_{on} \sin(\alpha_{on}/2)) = m_n h, \quad (40)$$

Where: $m_n = \pi / (\sin^2 \alpha_{on} \sin(\alpha_{on}/2)); \leftarrow = h / 2\pi.$

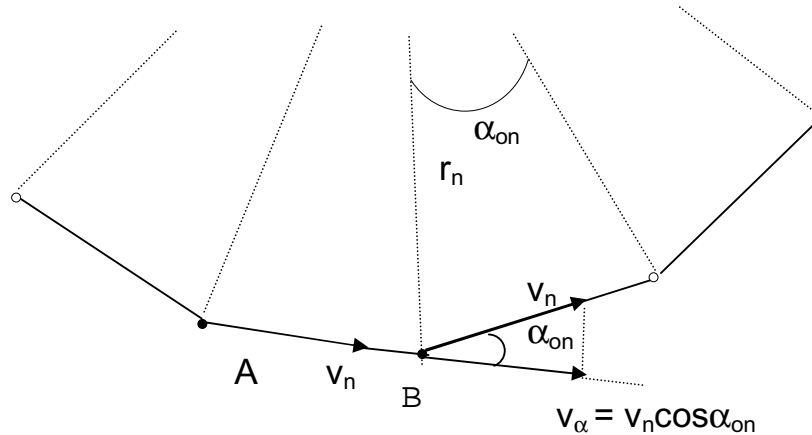


Fig.2. The motional orbit of electron only is broken line.

From (38), we see that in order to maintain an orbit of electron, an amount of a *least-effect* quantum is insufficient, it is a must to have k_n quantum h and thus according to (40), in case of orbit angular momentum M_n , it is needed to have m_n of quantum h. Then, as the motional direction can not be deflected on as small as wanted angles, thus, the motional orbit of electron can never be a curve, including circle, but only open or enclosed broken line. From fig.3, it is possible to say that the electron's motion on orbit is the “*pseudo uniform straight motion*”! That's so, it is clearly that the change of motional direction (angle acceleration) does occur but only at each discrete time t_n corresponding to the completion of action equal to h, “*almost*” in the entire time, it moves straight at velocity $v_n = \text{const}$ – without acceleration. The

graph presenting the instantaneous acceleration of electron in this case is also similar to the graph in figure 1b. That's why, according to the theory of electromagnetic field, it can not radiate electromagnetic waves.

IV. Conclusion.

1. The “**Least-action principle**” (LAP) is the result of the combination between the “Hamilton *least-effect principle*” and “*least effect*” – Planck h constant and the *action* – a similar dimensional quantity with *effect*. This new principle can be applied in the quantum mechanics.

2. According to this principle, the particles can not move increasingly or decreasingly in a continuous way, but move in “*recoil*” manner with steps of velocity jumping. The concept of instantaneous acceleration with fundamental particles is of no significance.

3. The “*physical wave*” do not exist! Particles also do not have “*wave property*” but “*similar wave manifestation*”. **There only exists a new property of particles that “their motion can only be deflected according to limited and defined “angle quantum” and that could not be as small as wanted”.**

4. In case, the electron's motional direction deflecting in atom occurs under the action of Coulomb force with equal *angle quantum* and their total deflection angle is always equal to multiple of 2π , thus, the orbit will be formed with sides of an equal polygon inscribed to a circle with radius r_n . From here, it is possible to draw out the orbit quantum condition of electron in atom. Furthermore, **the phenomenon without electromagnetic radiation of a electron on the its orbit can be fully described in the scope of classical electrodynamics without adding any other quantum condition.**

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